DO NOW

Let's take a look at Chapter 3 tests.

4.1 Extrema on an Interval

Definition of Extrema:

Let f be defined on an interval I containing c.

1. $f(c)$ is the minimum of f on I if $f(c) \le f(x)$ for all x in I.
2. $f(c)$ is the maximum of f on I if $f(c) \ge f(x)$ for all x in I.

The minimum and maximum values on an interval are called the: extreme values or extrema (singular form: extremum), of the function on the interval.

Called absolute (global) maximum or minimum.

Definition of Relative Extrema:

1. Relative maximum at $(c, f(c))$
   If there is an open interval on which $f(c)$ is a maximum
2. Relative minimum at $(c, f(c))$
   If there is an open interval on which $f(c)$ is a minimum.

The plural forms: maxima and minima

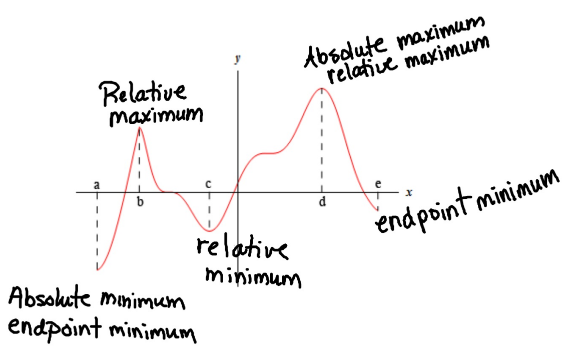
The Extreme Value Theorem:

If f is continuous on a closed interval $[a,b]$, then f must have both a maximum and minimum on the interval.

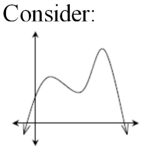
Types of Extrema on an Interval:

1. Absolute (global):
   highest/lowest point in the entire domain of the function.
2. Relative (local): "peaks & valleys"
   highest/lowest points in relation to nearby point
3. Endpoint:
   found at the endpoint of an interval

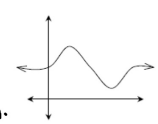
*See pg 204



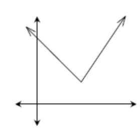
Consider:



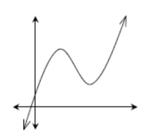
absolute max.
↳relative max
relative min.
no absolute min.



Absolute maximum
↳relative
Absolute minimum
↳relative



absolute min.
↳relative min.
No absolute max.



no absolute max.
no absolute min.

What can we say about the derivative at the extrema???

*Some derivative is zero
others does not exist.

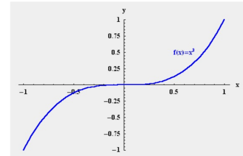
Critical Number: Let f be defined at c.

If $f'(c)=0$ or if f is not differentiable at c, then c is a <u>CRITICAL NUMBER</u> of f.

<u>THEOREM</u>:
If f has a relative minimum or relative maximum at $x = c$,

then c is a critical number of f.

Consider the following graph:



Where would critical numbers be?

$x = 0$

Are there any extrema there?

No...

* A maximum or minimum must be at a critical number.

HOWEVER – a critical number does not have to be a maximum or minimum.

<u>Guidelines for Finding Extrema on a Closed Interval</u>:

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1. Find the critical numbers of f in (a,b).
2. Evaluate f at each critical number in (a,b).
3. Evaluate f at each endpoint of $[a,b]$
4. The least of these values is the minimum and the greatest is the maximum.

Example: Find the absolute maximum and minimum.

1. $f(x) = -x^2 + 3x$ on $[0, 3]$

$f'(x) = -2x + 3$

$-2x+3=0$

$x = \frac{3}{2}$

left: $f(0) = 0$

critical #: $f(\frac{3}{2}) = -\frac{9}{4} + \frac{9}{2} = \frac{9}{4}$

right: $f(3) = -9 + 9 = 0$

$\boxed{\text{minimum: } (0,0) \text{ and } (3,0) \quad \text{maximum: } (\frac{3}{2}, \frac{9}{4})}$

2. $f(x) = -x^2 + 3x$ on $[-1, 3]$

$f'(x) = -2x + 3$

$x = \frac{3}{2}$

left: $f(-1) = -1 - 3 = -4$

critical #: $f(\frac{3}{2}) = \frac{9}{4}$

right: $f(3) = 0$

$\boxed{\text{minimum: } (-1,-4) \quad \text{maximum: } (\frac{3}{2}, \frac{9}{4})}$

HOMEWORK

pg 209 - 210; 1, 2, 9 - 13,
~~21 - 41 odd~~, 61, 63 - 65
24, 25, 27